JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 16, No. 5, September–October 1993

# Active Control Technology for Large Space Structures

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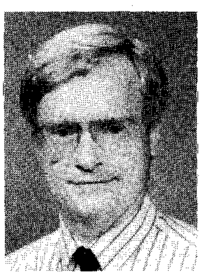
## I. Introduction

CONTROL design for attitude control and stabilization of the older generation of spacecraft consisting of a rigid bus with, at most, a small number of flexible appendages, has traditionally relied on rigid body dynamic modeling and control through one-loop-at-a-time, classical Nyquist-Bode techniques for single-input, single-output (SISO) systems. Spacecraft experience of the past two decades (see Table 1 and Ref. 1) indicates that control/structure interaction effects must be included in the design process, even for the traditional spacecraft configurations. Most importantly the advent of the Space Transportation System (STS) has sparked the conception of advanced space system configurations for which structural flexibility is not merely a matter of marginal impact but actually dominates projected system performance. It was recognized in the late 1970s that the highly flexible advanced space system concepts involving many sensors and effectors require the development of correspondingly advanced multi-input, multi-output (MIMO) control design techniques. Over the ensuing decade, the technical challenges of this development have stimulated the emergence of a distinct technological field which may be termed "active control of large space structures" (LSS).

The first question that comes to mind is: What makes a space structure "large" from the control point of view? The key ingredient in advanced system configurations is that there are many vibration modes within the frequency band of significant disturbances and the requisite control bandwidth. The situation depends on disturbance environment, structural flexibility, and performance requirements. For example, large ra-



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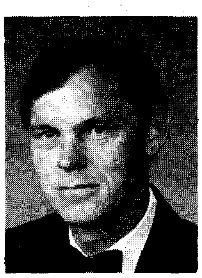

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Received March 29, 1992; revision received Sept. 18, 1992; accepted for publication Oct. 28, 1992. 

**Table 1 Control/structure interaction problems have occurred in every stage of space system life cycles with impact ranging from moderate to very serious**

| Year | Satellite | Control technique | Problem | Probable explanation for problem(s) |
|---|---|---|---|---|
| 1958 | Explorer 1 | Spin stabilized | Unstable | Energy dissipation in whip antennas |
| 1962 | Mariner | Attitude thrusters | Stable, gyros saturated | Solar panel flexibilty changed center of gravity |
| 1962 | Alouette 1 | Spin stabilized | Rapid spin decay | Solar torque on thermally deformed vehicle |
| 1963 | 1963-22a | Gravity stabilized | Vibrations excessive, but within specification | Boom bending due to solar heating |
| 1966 | OGO III | Reaction wheel | Excessive oscillations in attitude | Control system interaction with flexible booms |
| 1966 | OGO IV | Gravity stabilized | 1-2-deg oscillation | Solar radiation induced boom bending |
| 1969 | TACSAT 1 | Spin stabilized | Limit cycle, but within specification | Energy dissipation in bearing assembly |
| 1969 | ATS V | Spin stabilized | Unstable | Energy dissipation in heat pipes |
| 1972 | DMSP | 3-Axis torque balance | Solar array and controller interacted | Design was based on rigid body |
| 1973 | Mariner 10 | Attitude thrusters | Unstable roll, depleted fuel | Thrusters and gyros noncollocated with flexible panels between them |
| 1977 | Voyager | Attitude thrusters | Flutter of boom antenna | Thermal bending coupled with low torsional stiffness |
| 1982 1984 | LANDSAT | Spin stabilized | 0.1-deg oscillation | Thermal bending induced by entering and leaving umbra |
| 1984 | LEASAT | Spin stabilized | Orbit transfer instability | Unexpected liquid slosh modes |
| 1989 | Galileo | Spin stabilized | Schedule impacted, system identification added | Structural frequencies close to control bandwidth, model uncertain |
| 1989 | Magellan | Attitude thrusters | Design cost and schedule impact, redesign control law | Design of solar panels ignored attitude control system during solid rocket motor burn |
| (?) | Zenith Star | Attitude thrusters | Nonrepeatable modal frequencies for identical parts | Variability of materials and geometry |

dio frequency (rf) system concepts involve structures that are so flexible that many modes are contained in the control bandwidth even though neither performance requirements nor requisite control bandwidth are extraordinary. At the other extreme, many advanced solid reflector or optical systems involve relatively complex, stiff structures but the tolerances on pointing accuracy and surface accuracy are so severe that high-bandwidth control systems, again encompassing many vibration modes, are required.

Reference 2 gives estimates of the mode count (the number of vibration modes below a given frequency, plotted as a function of frequency) together with power spectral densities (PSDs) of the dominant disturbances for typical rf and optical systems. These and similar configurations for both NASA and DoD missions have $\sim 10^2$ vibration modes in the band of significant disturbances, require multiple control subsystems involving tens of sensors and effectors with control bandwidths encompassing at least some tens of modes. Thus, structural flexibility is not a marginal concern or secondary interaction to be coped with in ad hoc fashion, but a primary effect that dominates performance. The necessary development of systematic modeling and design tools for LSS control that has occurred over the past decade is the focus of the present review.

In conducting this review we place the emphasis on those developments that we believe have proved most useful to the LSS control design task and are most likely to succeed in actual implementation. Thus, our first priority in reviewing the literature is the discussion of methods we feel have the highest probability of near-term realization in hardware and results that appear to be the foundation of near-term improvements in control design methods. In fact, before composing this review, we agreed to imagine that we are actually building an LSS and that we are "on the spot" to do the controls design. Given this supposition: What problems must we address? What existing results do we seriously plan to use? In addressing these questions we hope to enumerate the major control regimes and their challenges, the available methods for meeting these challenges, and, finally, the gaps remaining to be filled to ensure successful implementation of this technology.

Considering mission scenarios across the board there are basically three performance regimes for LSS: rapid reorientation to point at a tracked object, tracking an accelerating target, and maintenance of geometric precision (surface accuracy, and/or optical quality) in the face of persistent disturbances (target accelerations and onboard broadband disturbances). When one lists the corresponding analysis and design tasks in their order of execution, the underlying logic of this sequence strongly motivates the treatment of modeling issues, model acquisition (system identification) and model simplification *first*, of rapid retargeting and tracking *second*, and of vibration control *third*.

Accordingly, Sec. II reviews those aspects of model formulation, model simplification, and system identification that form the basis for the control design activities. Section III reviews developments in retargeting and tracking control wherein the principal challenges spring from the urgency (implied by the need for rapid maneuvers) and the nonlinearities manifested in both plant and control. Active vibration control to suppress steady-state jitter and postmaneuver transients is reviewed in Sec. IV. Here, the technical challenges arise from the potentially large number of inputs and outputs and the unprecedented complexity of the plant dynamics. The efficacy of these developments is illustrated in our review of hardware implementations and experiments in Sec. V.

Here we emphasize post-1982 experiments, involving complex built-up structures, that have proceeded full term to yield experimental test results. Finally, remarks on future needs and desirable directions are presented in Sec. VI.

In a sense the present review forms a sequel to the 1984 review of Ref. 3. It is fair to say that since that time the LSS controls field has made great strides toward applicability and maturity. It is hoped that our discussion, particularly the experimental results in Sec. V, will fully illustrate the efficacy of advanced active control methods for LSS and adequately portray the opportunities these methods open up for LSS development.

## II. Modeling and System Identification

### A. Types of Mathematical Models

The first step in control system design is to obtain a mathematical model of the system to be controlled.[4,5] The control system designer normally uses a number of models, each for its own purpose. He employs a complex evaluation model to predict control system performance. A simpler model is used for the control law design process. He might also use a number of different simplified models of the same structure for different control objectives, such as structural vibration control, pointing control of an antenna substructure, and rapid slewing or retargeting of the spacecraft structure. The form of such a model can be chosen based on physical laws together with some inevitable idealization, or it can be a form appropriate for use with some field of control theory, or it can be a model that is appropriate for some specific system identification technique.

Unfortunately, the modeling of large flexible spacecraft is sufficiently complex that it is not obvious what type of physical model to use—the decision already involves some model reduction issues. Models based on physical laws include a pure continuum or distributed parameter model; a wave propagation model; a multiple rigid body model with spring and damper interconnections modeling distributed elasticity; a hybrid coordinate model including rigid body and distributed parameter models perhaps using assumed modes; a second-order matrix model with mass, stiffness, and damping matrices; vibration frequencies and mode shapes; and finally, a large scale finite element model. Physical models are important because 1) they may give better physical intuition for the designer; 2) they can predict the behavior of the whole structure, not just predict behavior at measured points, which can be important in evaluating control system performance; and 3) they can be used to predict the effects of environmental disturbances on control system performance.

Control theory and the preponderance of work in the system identification field emphasizes realization theory, which produces a model whose only requirement is to closely match input-output data without any consideration given to physical laws. A realization is a reasonable model to use when the only variables that one is interested in controlling are measured variables. The models appropriate for various fields in control theory include 1) certain simple continuum models for which a reasonable distributed parameter control theory exists, 2) standard first-order matrix linear differential equations, 3) first-order matrix linear difference equations, 4) Laplace transfer function matrix form, 5) $z$-transfer function matrix form, and 6) matrix auto regressive moving average (ARMA) model; in addition, one could also perform control design based purely on 7) frequency response plots alone. The control system designer must pick the model type based on the availability of control methods associated with the type, the difficulty of their use, and their adequacy for the problem. A similar array of model choices applies to identification. Either the same model type should be used for both identification and controller design, or it should be easy to convert the model types.

The most natural model is different for different communities. In the strucures community a finite element model is the most natural for computer model generation of a complex structure. For physical understanding of a structure's behavior, the structural dynamicist wants modal frequencies and mode shapes. On the other hand, multivariable control system design is most naturally done using a modern state variable linear differential equation model. But control system implementation is most naturally accomplished in digital control form with all of the added considerations of aliasing, hold devices, and possible pre- and postfiltering. System identification is most easily accomplished with ARMA models for which the identification problem is a linear one, whereas state variable models and differential equation models usually produce nonlinear identification problems.

The form of the model can also influence identifiability, i.e., the ability to uniquely determine the model parameters from data. The large number of parameters in a finite element model will generally be underdetermined from data. It is shown in Ref. 6 that for a mass and stiffness matrix model one needs a large number of sensors and actuators to be guaranteed in advance that one will be able to identify all of the elements from data. If, on the other hand, one considers it reasonable to model the structure as a flat plate or a uniform beam, the number of parameters needed would be very small. From this small number of parameters of the idealized continuum model, all of the elements of the mass and stiffness matrices are determined, as well as the parameters of a finite element model. Several authors have studied the use of continuum models for such reasons, e.g., Juang and Sun.[7] It is a form of physically motivated model reduction: a reduction in which the number of parameters of the model is reduced, not the dimension of the model.

### B. Contrasts Between Modal Testing and System Identification

Like all control applications, the LSS problem involves the interaction of the control field with another field, in this case structural dynamics. Both fields have well-developed identification methodologies, and the two are very different. Identification in structural dynamics, usually termed modal testing, refers to experimental determination of modal vibration frequencies and their damping factors, and with somewhat less emphasis, mode shapes and modal participation factors. Modal testing has a long history of generating test methods finely tuned to structural vibration problems, with considerable sophistication having been developed in experimental procedures and handling of data. See for example the bibliography in Ref. 8 and the overviews in Refs. 9–13. The feature article by Juang and Pappa[14] gives a particularly good overview with a good bibliography. On the other hand, the identification methods from control theory, normally aimed at generating an input-output realization, have a very rich and sophisticated theoretical foundation. But they are general purpose in nature, so they are not finely tuned to structural dynamics problems and do not benefit from the vast experimental experience in modal testing. The LSS problem pushes the state of the art for both fields and will require a combined effort of researchers in both disciplines. To address the LSS challenge, there is a need to foster full cross fertilization to fill the gap between these fields.

Historically, the modal testing community has emphasized modal frequencies and damping without getting all of the information needed by the control system designer. He has a number of basic needs in testing. Once candidate actuators and sensors have been chosen together with their locations, he is interested in input-output tests with these sensors and these actuators. The actuators, especially, are likely to have their own dynamics, which are not the usual modal vibration dynamics. By contrast, modal testing uses shakers instead of control system actuators, and often uses accelerometers as the only sensors. In lightly damped systems, an accelerometer output of an oscillation looks like the negative of a position measurement. However, unless the only objective of the control system is to suppress vibrations, direct position measurements are needed for control. This too makes for a nonstandard modal testing problem.

Often modal testing simply observes the free decay of vibration modes after some excitation, without recording the actual excitation function or where it is applied. The resulting information is incomplete for the control system designer, since no information is obtained concerning the input influence matrix. Modifications of the common test procedures are required to monitor the input time histories and their locations for this purpose. In the case of accelerometer measurements, there is an additional output matrix which must be identified corresponding to the direct feedthrough of input forces to output accelerations in the standard modern control measurement equation. Determination of this direct feedthrough term is not part of standard modal testing.

The control system designer may want more detailed testing as well. Additional sensors besides those used in feedback control might help him evaluate the complete control system performance, and additional actuator locations might help him predict the effects of disturbances. We conclude that fundamental extensions of the standard modal testing procedures are needed to address the LSS problem.

Some description of modal testing procedures is instructive, including the considerations and the difficulties involved. In spite of the theoretical foundation underlying modal testing, it still remains somewhat of an art. Pappa[15] gives a nice summary. Many aspects of modal testing appear simple, even trivial, in theory, and yet are both surprisingly difficult in practice and very important for successful identification, particularly for the complex structures considered here. Successful use of identification techniques with practical structures is observed to be problem dependent, technique dependent, and user dependent. Data quality is very important; many identification methods are significantly affected by subtle data irregularities such as slight nonlinearities, especially with high modal density, inconsistent data sets from tests conducted at different times, instrumentation errors such as brief saturation, and extraneous disturbances. Modern techniques can decrease the inconsistencies in data,[16] and inputs optimized based on nominal models can improve conditioning of the problem.[17] Pretest analytical-computer predictions are an essential part of modal testing for the purpose of developing a test-analysis model. This is used to pick the set of physical coordinates at which measurements will be made in the test, to study possible shaker locations, and used to compute orthogonality of mode shape results using its mass matrix. It can also be used for spatial filtering and for enhanced data processing. Exploratory experimental studies before the start of actual data acquisition are an important part of modal testing, to optimize excitation and data collection parameters. Important information obtained as listed in Ref. 15 includes the frequency bandwidth of interest for selection of appropriate filters, the effectiveness of various excitation locations and directions, the modal density that influences the requirements for multiple-input excitation and shaker locations, reciprocity checks to optimize data consistency from separate excitation sources, amplifier range settings avoiding both under- and overload situations, calculation of driving-point frequency response functions to incur high-quality reference measurements, the degree of nonlinearity for selecting appropriate excitation levels, the approximate level of damping influencing the data collection time requirements, background noise level to establish adequate data averaging parameters, quick identification of rigid body or other simple mode shapes to insure proper orientation and functioning of all sensors, repeatability of results to examine experimental stationarity, and rattles or other mechanical trouble areas to be fixed or eliminated, if possible, prior to data collection. Even with this preparation, modal testing rarely proceeds in a routine manner, but rather various secondary testing is performed to insure accuracy and completeness. Typical items include linearity checks, detailed mode shape definition in poorly understood areas using roving sensors, increased focus on modes found to be larger contributors to loads or responses than predicted, comparison of identification results obtained using two or more different techniques, modification of shaker locations or directions to enhance excitation of particular modes, and ad hoc tests to understand better any modes or other phenomena observed in the test but not predicted by analysis.[15] Such secondary testing often contributes in a significant way to the success of modal testing identification. The preceding description of modal testing makes it clear that the experimenter's experience plays a major role in successful identification in structural dynamics—both general modal testing experience and, especially, previous experience in testing similar structures. One implication is that the use of automated on-line identification for the purpose of adaptive control may be too simplistic to be successful.

Since the mid-1960s system identification has been an important part of the control field, and the identification methods when adapted to the modal identification problem often bear little resemblance to those used in the structural dynamics community. The reader is again referred to Ref. 14 for an overview and bibliography. The methods divide into frequency and time domain methods. The frequency response function can be obtained using the sinusoidal transfer function analyzer. It is robust due to its ability to reject low-frequency drift and harmonic distortion from nonlinearities, but it requires long test times. This corresponds to sine-dwell methods of modal testing, which are now mostly replaced by quicker methods. Other frequency domain approaches use digital spectral analysis and fast Fourier transform methods, which are usually applied on data blocks and the resulting spectral estimates averaged. The frequency response function is a nonparametric system identification result which limits the set of control design approaches that can be applied directly. There are many available methods that can closely curve-fit such data to obtain a transfer function or modern control parametric representation, but experience with fitting of modal frequency response functions for complex structures shows that the resulting parameters, particularly the mode shapes, can be unreasonable in light of physical expectations. Time domain identification is better matched to many control design methods, and can take many forms.[14] Nonstochastic methods usually use least squares methods, and instrumental variables can be introduced to address the bias problem. Generalized least squares can estimate the noise parameters as well as the system parameters. Other methods include the extended least squares and stochastic approximation and the extended Kalman filter. One can demonstrate a close relationship between all of these methods. Another class of approaches obtains parameter estimates by maximizing a likelihood function. There is a strong theoretical basis for all of the approaches.

### C. Development of Large Space Structures Identification Algorithms

Not all of the different identification methods coming from the structures and controls communities have received substantial attention in the attempt to address the LSS control problem. And experience with others demonstrates that the methodology of application can be very important to the effectiveness of the method. Maximum likelihood identification of modal parameters is studied in Ref. 18 and their recursive least squares estimation is studied in Ref. 19. Ladder or lattice filtering is a recursive method for solving the linear least squares problem, which can be thought of in its state space form as corresponding to using a state vector with a diagonal covariance matrix.[20,21] References 22–24 apply it to flexible structures. Because of the complexity of the flexible structures identification problem, methods requiring iterative solution of nonlinear equations with possible problems of local minima do not seem appropriate. What is needed is a method that gives accurate results that one can trust.

A very good discussion in Ref. 25, giving comparisons based on realization theory, shows the relationships between various methods that have appeared in modal testing over the last 10–15 years. The least squares regression technique for identification with discrete-time dynamic models has been used in the control community for more than two decades.[26] The same technique was rederived and further developed for modal parameter identification.[27,28] It is shown in Ref. 25 that the method does not produce a minimal realization and can rely on noise to eliminate rank deficiency problems. The polyreference technique developed in Refs. 29–31 using frequency-response functions for identification of modal parameters from multi-input, multi-output data, can be thought of as a realization in the form of a canonical representation that is not necessarily minimal. It is shown in Ref. 25 that the approach can give either a controllable canonical form or an observable canonical form.

The eigensystem realization algorithm (ERA) developed by Juang and Pappa for modal parameter identification is an example of the type of cross fertilization needed between the control and structures communities. It is an outgrowth of the Ho-Kalman minimal realization theory,[32] and performs modal parameter identification and model reduction from test data.

A concise presentation of the underlying theory is given in Ref. 33. The relationships to polyreference and least squares regression are shown in Refs. 25 and 34. Alternative forms of ERA perform identification in the frequency domain (ERA-FD),[35] or in a recursive form,[36] or based on data correlations (ERA-DC).[37] These methods use impulse responses either obtained from fast Fourier transform inversion of input-output data or from a time domain computation from the input-output data. Very similar mathematics is generated in the stochastic case viewing the state space in the form of a prediction space according to Ref. 38. The $Q$-Markov cover approach of Refs. 39–41 is also in the same family of methods, as is much of the work of Staar, e.g. Ref. 42, and the method in Ref. 43, which allows direct use of data other than measured or derived impulse response or white noise response.

Numerical tests suggest that ERA-DC gives the best performance among ERA, ERA-DC, $Q$-Markov, and the method of Ref. 43, when compared in terms of $H_\infty$ norms of identification error.[44] ERA-DC has ERA as a special case and, hence, is always able to match its performance. $Q$-Markov cover required use of significantly longer data strings to approach the identification performance of ERA-DC. It was shown that when used on noiseless data $Q$-Markov cover gives only asymptotic convergence to the true parameters as the number of data points is increased. The method of Ref. 43 seemed to require careful choice of inputs to give good performance and suffers from increased dimensionality.

In practice it is important to have some way of evaluating the uncertainty in identification results. Different methods apply to different identification approaches, e.g., maximum likelihood, Cramer-Rao bounds, and confidence tests. One method is to numerically superimpose a known mode onto the data and test the ability of the algorithm to accurately identify this known mode.[45] ERA practitioners, in addition to using certain confidence criteria associated with the method, often make Monte Carlo runs with data sets altered by adding representative pseudo-random noise of a level consistent with the expected measurement noise level.[46] Observing the resulting amount of scatter of the identified modal parameters shows the sensitivity of the results to measurement noise and indicates how much confidence one should place in each identified parameter. In Refs. 33 and 47–49 this procedure is replaced by an analytical perturbation analysis of the nonlinear ERA, or ERA-DC algorithm, and additional error level information and order determination information is also obtained. The linearized relationship between measurement noise and identified parameters allows one to compute the variance of the identified parameters; and within the accuracy of the linearization, one can establish confidence intervals about the identified parameter values. Estimates of bias are also obtained as a second form of identified parameter uncertainty. These criteria can tell the user when an identified mode is due to noise and when it is a true mode in the system.

These methods are also useful in assessing the true system order. The variance and bias of the singular values are seen to be particularly sensitive measures of true system order, which can distinguish true order when noise levels are too high for the singular values themselves to indicate the true order.

In each method there is a set of parameters of the identification routine that the user must choose, such as the amount of data to use, the sample rate to use, the shape and dimension of the Hankel matrix, the summation parameters in ERA-DC and $Q$-Markov cover, the amount of truncation of singular values, and the choice of any further truncation of identified modes. The perturbation results of Refs. 33 and 47–49 hopefully will decrease the importance of user experience by giving immediate feedback to the user concerning the relative desirability of the various parameter choices as he adjusts them.

In conjunction with these identification methods, improvements in identifying lightly damped, large dimensional structural dynamic systems have appeared based on first determining the Markov parameters of an observer and, from this, identifying the system using ERA or ERA-DC.[50] The approach compresses the data and alleviates the dimension problem in the identification.

Another way to characterize the uncertainties in an identified model is to identify the measurement noise level and the Kalman gain at the same time as the system dynamics. Methods similar to the mentioned observer identification accomplishes this direct identification of the Kalman filter from input-output test data, and this can be directly useful for the control system designer, as well as a step in the system identification process.[51,52] This is an example of improvements at the interface between system identification and control system design. Additional improvements for relatively short data lengths can be produced by whitening the residuals.[53]

### D. Research at the Interfaces

Much of the important research in control of LSS has aimed at improving the interfaces between different fields.

#### *1. Modal Testing/Control Interface*

Modal testing procedures will have to be expanded and focused to obtain the information needed by the control system designer. Many of these needs have already been stated above in Sec. II.A. The control system designer must determine additional needs which can be different for different approaches to control law design. For example, as discussed earlier, it is important to have methods that can indicate the degree of uncertainty in identified parameters, since this can form the basis for many decisions in the control law design, including the robustness requirements. Designing a controller by the $H_\infty$ approach requires that the uncertainties in the model be characterized in a specific way, see for example in Ref. 54. There is a tradeoff between effort expended for good identification and effort spent at producing controller robustness with the associated compromise in control system performance. A unified control-identification methodology involving the uncertainties of the identification is needed.

Skill and experience is still an important ingredient in successful modal testing and even in optimized use of identification algorithms such as ERA-DC. Eventually we would like methods that can work routinely. Perhaps closer interaction between the modal testing and control communities can produce such improved methods by somehow characterizing and automating the decision process used. Also, improved estimates of modal damping would be particularly helpful since control law effectiveness is usually highly sensitive to system damping.

#### *2. Identification/Model Reduction Interface*

Control law design is often limited in the dimension it can handle and needs a restricted order model for design and implementation, but an accurate model for evaluation of expected control system performance by simulation. As discussed in Sec. IV, the controller dimension problem can be handled by model reduction before control law design, by designing a large order controller and reducing it, or by designing a fixed-order controller for a high-order model. But in all cases it appears that for control of large flexible structures, one needs some initial model reduction before the start of the control design process. Historically, the structures community has performed model reduction based on truncation of higher order modes, although there are other approaches such as that of Ref. 55. The control community has a richer set of reduction methods such as reduction of generalized Hessenberg representations,[56] balanced forms, modified versions of balanced forms,[57,58] or more general degree of controllability/degree of observability concepts,[59,60] component cost analysis, optimal projections, etc. Many of these are discussed in Sec. IV.D. These methods are progressing toward choosing the form of a reduced-order model based on the use of the model. Some form of integration of identification with the control objective and the disturbance environment modeling is called for, to assess what modes need to be in the model. One step in integrating the identification and control objectives is given in Ref. 61 where identification is made on filtered data in a prescribed frequency range and then combined with frequency weighted cost functionals to obtain a consistent unified modeling and control synthesis.

*3. Realization/Physical Model Interface*

One purpose for using a model based on physical laws is to deduce system behavior at points other than measurement points, which can be important, for example, in predicting antenna performance. The physical law then fills in the relationships between the measured quantities and the other desired quantities. New sensors may alleviate this problem, for example, with a large number of mirrors on the structure and many laser ranging signals, so that measurements can be made at a large number of points and thus obtain good mode shape information. But the physical model is also important to be able to predict the structure's response to environmental disturbances such as thermal distortions, gravity gradient distortions, aerodynamic effects, and internal moving parts. The situation is complicated by the fact that identification methods for physical parameters are generally much more difficult than creating a realization. Chapter 6 of Ref. 25 gives a nice discussion. The physical parameters could be coefficients in a set of partial differential equations, but the complexity of typical spacecraft structures makes this less natural in practice than in theory. Treating the properties of finite elements as the physical parameters is natural in practice, since nearly all modeling for complex spacecraft uses finite element analysis. The difficulty here is that the adjustment of finite element modeling of a continuous structure is an underdetermined problem,[62] and one needs to add some extra physically motivated limitations.

*4. Control Theory/Implementation Interface*

Recent laboratory tests for control of LSS have made it clear that the limitations and nonideal behavior of the sensors and actuators can have a determining influence on control system performance. Control law design must take this behavior into consideration.

### E. Some Testing Challenges

*1. Optimized Test Signals*

Optimization of the excitation input in structural dynamics testing is a relatively untapped source for improving test results, which may become more important in the on-orbit test environment where short testing times would be desirable. Some work has been done in the structures community,[17] and there is a substantial theory developed in the control literature.[25,63,64] In spite of the existence of the theory, the implications for structural dynamics testing have not yet been fully understood and exploited.

*2. Testing of Nonlinearities*

Nonlinearities of joints may require special testing, and the implications for control law design and performance are not clear, since there are relatively few design approaches that can directly address nonlinearities in complex systems, see Refs. 65 and 66 and the references therein. Also, testing of nonlinear large angle slewing performance will ultimately be made. The simulation of such maneuvers is perhaps the most computationally intense mechanics problem in the aerospace field, with various methods for improving the computational situation being given in the references in Ref. 4 and with further improvements needed. Perhaps one should get beyond modal thinking in these problems since modes do not exist in a mathematically rigorous sense in such transient situations,[67] with gyroscopic, and nonlinear centrifugal and Coriolis effects present.

*3. Testing Difficulties in a 1-g Environment*

LSS will generally have insufficient structural integrity to be constructed on Earth in the 1-*g* environment for purposes of testing. Ingenuity is then required to find ways to validate the design before launch. One possibility is to test components that are sufficiently small that they can be tested under gravity. Such testing is often done by subcontractors before integration of the spacecraft, so that the problem of combining such subassembly models is one that has received attention. Unfortunately, the linking of the models can easily tax one's computational facilities, and this pushes one into a different version of model reduction,[55] or into component synthesis methods that can be broadly classed as fixed, free, or loaded interface methods; see the references in Ref. 4. A second approach is to make use of scale models in testing; see the references in Ref. 4. In each case, there are requirements for suspension systems that do not interact with the low-frequency modes. Making the suspension cables longer can easily be prohibitive with the low vibration frequencies of LSS and more innovative experimental simulation of the zero-*g* environment are needed.[68,69]

*4. On-Orbit Testing Challenge*

Because of the inability to perform full system tests on the ground, on-orbit testing must be considered for purposes of tuning the control system. These could be off line, or on line for adaptive control use, but the system dynamics (excluding any system failures) are not changing sufficiently fast to demonstrate a need for on-line identification, and the identification problem for LSS has proved to be sufficiently difficult that an on-line automated procedure might not be trusted (for now).

Planning for on-orbit testing of LSS poses various challenges.[15] One needs to answer questions such as: should certain modes be considered more important to identify than others, on what basis, and how should one tailor experiments for such a purpose? If the on-orbit tests are so tailored, how does one insure that they are simultaneously tolerant of all realistic errors in the pretest predictions? How many sensors of what types are needed, and where should they be placed? How can one foresee, or eliminate the need for, the extra ad hoc experimental studies that modal testing people perform on complex structures to know how to perform experiments that obtain good results? The success of ground-based identification often hinges on decisions made as a result of such tests. Consideration must be given to the presence of environmental disturbances. These must either be considered negligible during the on-orbit tests so that their effect on the system behavior can be ignored, or they must somehow be measured or otherwise known for the structural dynamic system being identified. Another discussion of on-orbit identification appears in Ref. 70.

On-orbit testing implies that the sensors and actuators on the spacecraft must serve a dual purpose: both for feedback control (and perhaps state reconstruction) and for identification. Of course, if needed one could have additional sensors and maybe even actuators for the identification task. Sensor and actuator location for control is studied in Refs. 59 and 60 by degree of controllability and degree of observability, and in Ref. 71 by component cost analysis, to name only a few of the contributions in this problem area. A preliminary study of the consistency or inconsistency of sensor and actuator placement for control vs for identification is given in Ref. 72. In planning the on-orbit tests, one would like to be guaranteed that the data that will be collected in orbit with the chosen sensors and actuators and their chosen locations will, in fact, be sufficient information to identify the structural dynamics model. This identifiability issue is studied in Ref. 6.

## III. Rapid Retargeting and Tracking Control

### A. Introduction

A significant fraction of the missions envisioned for the next generation of spacecraft require that a large flexible vehicle undergo rapid nonlinear maneuvers followed by precision pointing and shape control of a substructure. Designing control laws for this class of spacecraft is difficult because the governing equations are usually nonlinear, of moderate to high dimension, and may involve significant internal and external uncertainties. As a result of the coupling between rotational, translational, and vibratory motion and as a consequence of geometric/kinetic/constitutive nonlinearities, one usually cannot blindly superimpose vibration suppression control on a maneuver law designed assuming the structure is a rigid body. Furthermore, we are often confronted with the need to (formally or by ad hoc methods) find a control design that achieves a judicious compromise between competing performance measures, such as: minimizing maneuver time, minimizing control energy, and minimizing sensitivity to model errors; these objectives are inherently competitive and must be compromised in the presence of actuator saturation bounds, sensor/actuator bandwidth limitations, and state-space inequalities such as maximum bending moments.

At this stage of theory and algorithm development, rigorously formulated control design tools are available only for approximate and/or incompletely stated nonlinear structural maneuver problems. Solution for a practical control design can still proceed using available methods, but most often the lack of rigorous closure requires augmenting the formalized design methodology with a combination of parametric studies, pragmatic judgement, and experimental research. The absence of a complete, end-to-end, mathematically and physically rigorous methodology for maneuver control design is a source of frustration to some and excitement to others. In spite of the need for further theoretical and computational developments, it is of importance to observe that the methodology is sufficiently well developed that good designs and hardware implementations can presently be achieved. This judgement is based on our evaluation of the recent research literature, especially that documenting joint analytical/experimental studies.

In reviewing the literature, we have given special attention to the subset of control design methods that have been successfully implemented in ground-based experiments. We have also given high priority to methods that have been studied analytically or numerically, including detailed consideration of the necessary sensors, actuators, and modeling issues associated with hardware implementations. Where we find many closely related papers by the same school of investigators, we have referred to key papers and most recent papers; these decisions are subjective but necessary to keep the length of this review reasonable. In a single review paper of this length and scope, it is of course impossible to be truly comprehensive and refer to every paper published in the past two decades. The key objective underlying our approach is to discuss the literature that we feel provides the best foundation for well-conceived, near-term implementations. To provide a more complete perspective, we selectively broaden this discussion to consider literature dealing with more basic mechanics and control theory that we feel underlies the full understanding of the current methods and research trends.

Before discussing the optimal maneuver research literature, it is important to focus on the major technical dilemma that is implicit in this family of problems. A "frontal assault" to design a nominal open-loop maneuver using optimal control theory leads to a nonlinear two-point boundary-value problem (TPBVP) of moderate to high dimensionality.[73-75] Since we are usually seeking near-minimum-time maneuvers, this obviously implies a sense of urgency in solving the TPBVP! Moreover, since real-time, on-board solution of the typical TPBVP is not within the power of available on-board processors, we are immediately presented with a dilemma: Since we cannot solve for optimal, high-dimensioned, nonlinear maneuver controls in real time, we must 1) find ways to use precomputation (e.g., interpolation from a precomputed and stored "extremal field map" of optimal maneuvers between all conceivable boundary conditions), and/or 2) introduce approximations that yield a suboptimal TPBVP that can be solved in real time. Of course, the optimization of a tracking-type feedback law to null departures from a nominal open-loop maneuver cannot proceed rigorously until the open-loop maneuver is established, and substantial computation is often required to optimize the feedback law. The research literature reviewed next presents methods that do not uniformly address these important implementation issues, however, our discussion and evaluations will be based in part on our assessments of methodology maturity vis-a-vis implementation.

### B. Smooth Nonlinear Maneuver Control via Pontryagin's Principle

A direct application of the optimal control necessary conditions leads to a nonlinear TPBVP whose solution is an optimal maneuver satisfying specified terminal constraints. Formulations and solution algorithms are presented in Refs. 73 and Refs. 76-78. The classical necessary conditions can also be used to establish feedback forms for the optimal control, as carried out for linear feedback,[79-81] and nonlinear feedback.[82] None of these developments, however, have any guarantees of global stability, and they do not represent attractive (vis-a-vis high dimensioned implementations) ways to compromise between minimum time, minimum vibration, and insensitivity to model and implementation errors.

### C. Toward Globally Stable Near-Minimum-Time Control of Flexible Body Maneuvers

Perhaps the first inclination an engineer might have when seeking a near-minimum-time rotational maneuver law for a flexible spacecraft is to ignore nonlinearities, cross-axis coupling, and flexibility. Under these severe approximations, the problem reduces to the equivalent of three uncoupled rigid body maneuvers, and the well-known minimum-time solution is available in the form of independent bang-bang control[73,74] applied to each axis of rotation. Unfortunately, only for very rigid vehicles and small angle maneuvers do these approximations yield feasible controls. Because of the shortcomings introduced by these approximations, several investigators have extended the classical rigid body analysis to consider flexibility effects.[83-90] In particular, the paper by Singh et al.[84] represents the most complete results for multiple-switch, bang-bang torque histories for exact control of an arbitrary number of flexible modes of a hub-appendage structure undergoing a planar rest-to-rest maneuver. Unfortunately, these results do not appear to be generalizable to nonlinear three-dimensional rotational maneuvers, and the switch times for the linear planar motion case are highly sensitive to the mathematical model of the structure, especially when controlling higher modes. Also, implementation errors associated with sensor and actuator dynamics need to be addressed in the future development of these ideas.

The discontinuous switching class of controllers inherently introduce unintended excitation to the higher frequency, less well modeled dynamics of the system; this naturally motivates the quest for torque-shaped controls that exhibit a high degree of smoothness. Optimal control approaches based on minimizing the integral square of higher control derivatives are introduced in Ref. 73, and highly successful laboratory experiments are reported and comparison with conventional approaches are reported in Ref. 91. Sine-versine, quintic splines, smoothed approximations of the sign function, and numerous other ad hoc parameterized input profiles have been used[92-97] to establish open-loop reference trajectories. Typically, a feedforward torque is computed that would make a rigid body follow the parameterized path, feedback tracking laws are introduced[95-104] to null the rigid and elastic departures from the parameterized motion. Based on simulation studies, the parameters of the ad hoc smooth path can be adjusted to achieve a compromise between maneuver time, some measure of tracking and/or vibration errors, and the robustness with respect to model errors. These approaches are in a sufficiently mature state of development that they can be applied to problems of moderate complexity.

One key issue in the large angle maneuver problem is that the structure of the control law for the large angle portion of the maneuver is not usually optimal for the terminal pointing and vibration arrest regime of the maneuver.[73] Thus "handoff logic" to detect entry into the terminal regime is required; some recent control laws have been developed wherein there is a seamless continuous transition from the large angle regime of the maneuver to the end game controller. In Ref. 96, for example, a near-minimum-time tracking law was formulated, studied numerically, and applied in a successful laboratory experiment with a highly flexible four body structure undergoing 40-deg rest-to-rest maneuvers. The tracking law is formulated such that the same gains that operate on the tracking errors en route operate on the terminal errors after the reference trajectory splines into the target state (or osculates with the accelerating target's trajectory, as may be appropriate). In Ref. 94, similar ideas are successfully implemented for experiments with a two-body flexible manipulator experiment; excellent agreement between the mathematical model and experimental results were achieved.

An important recent development is the successful marriage between Lyapunov stability theory and the design of large angle maneuver control laws for distributed parameter systems. The historical developments[105] for globally stable control of rigid body maneuvers were recently extended[100,106] to consider nonsingular attitude parametrizations and in Refs. 96, 97, and 107 these ideas have been successfully applied to design globally stable controllers for both rigid and distributed parameter systems. The control law of Ref. 107 is a constant gain output feedback law that guarantees global stability for large angle planar maneuvers of a hub-appendage model; the stability proof is not subject to truncation/spillover questions, because the stability arguments are based directly on the partial differential equation description of the system. Furthermore, results from successful experiments are reported. However, this approach requires gain scheduling to effectively carry out both large angle maneuvers and terminal vibration control. Extending these ideas, Refs. 96 and 97 present a control law design methodology that permits a user-designed smooth reference trajectory to be introduced; the resulting feedforward/closed-loop tracking law for large angle maneuvers still has guaranteed Lyapunov stability-in-the-large based on the governing system of nonlinear partial differential equations. Successful laboratory experiments are also reported in Refs. 96 and 97.

### D. Other Ideas for Large Angle Maneuver Control Law Design

There are many other approaches in the recent literature[108-118, 202-204] for designing control laws for large angle maneuvers. The so-called *exact linearization* methods[109,110] are based on introducing nonlinear transformations on the state and control variables to obtain a transformed system of strictly linear differential equations. Because the transformed equations are linear, it is possible to directly apply optimal control theory or other control design methods to construct an explicit linear controller in the transformed space. The control law designed in this linear space is then inverse transformed to obtain the physical controls, which as a consequence of the nonlinearity in the transformation is obviously a nonlinear control law. There are many difficult unanswered questions regarding this approach. First of all, the definition of "optimal" is a key issue, because simply designing a linear quadratic regulator in the transformed space, as is commonly done, cannot be justified on physical grounds. Second, the transformation is not unique and thus has an ad hoc flavor; for most cases in the literature, the transformation amounts to grouping all nonlinear terms together with the physical control and renaming this collection of terms the transformed control variable! There is no guarantee that the resulting controls designed in the transformed space will be physically realizable (e.g., due to control saturation, especially if the transformation contains singularities, because the computed physical control becomes infinite at the singularities). The exact linearization methods need to be further developed to address the issues just raised.

In Refs. 108 and 111-118, *variable structure* controllers are discussed in which arbitrarily fast control switching is permitted to force outputs or selected states of a system to slide along prescribed phase space trajectories in the vicinity of the target state. These ideas, motivated by classical switching controllers such as bang-bang control, have been shown to lead to attractive theoretical robustness because the control law and stability arguments are often free of physical parameters. The extension/application of these ideas to infinite (or high) dimensioned systems needs to focus on the consequences of possible spillover of energy into degrees of freedom not explicitly forced to slide to the target state and on the fact that physical actuators have finite bandwidth and thus cannot switch with arbitrarily high frequency. The chattering phenomena implicit in these *sliding mode* control approaches needs to be studied further vis-a-vis power/energy consumption and reliability.

### E. Implementation Issues

Real sensors and actuators have dynamics, lags, bandwidth limitations, saturation and noise characteristics that must be considered carefully to successfully implement any control law; this is certainly true of the sensor/actuator systems for large maneuvers of flexible vehicles. Although simply including sensor and actuator dynamics in the description of the dynamical system is possible, the usual problem of structural model errors are compounded with sensor/actuator model errors to significantly complicate the system identification and control design problems. There is no proven methodology that completely eliminates the theory/practice gap and achieves implementations in a cookbook fashion. Even though some few applications do work on the first attempt, these remain the exception. Improved methods for using identification-derived models promise to decrease the elapsed time for on-line tuning.

For typical attitude control actuators [e.g., control moment gyroscopes (CMGs)], the traditional path leads to an attitude control law which generates a commanded control torque. We often find that 1) the commanded torque cannot be exactly realized, or 2) it can be realized by an infinity of actual control commands. These problems are especially interesting for CMGs, because of the high frequency of adoption of CMGs for large spacecraft. The analysis is usually partitioned between finding a "commanded control" and an auxiliary "steering law" required to cause all of the reaction and gyroscopic terms arising due to the CMGs to sum to yield an effective interaction torque equal to the commanded torque. This approach is quite analogous to the "exact linearization" idea, and as in the exact linearization approach to control law design, one must take care to avoid singularities. A singularity occurs when the gyroscope gimbal angles assume certain configurations with respect to the magnitude and direction of commanded torque vector. The encounter of these singular configurations is a function of the particular fashion in which the non-unique gimbal torques are determined by the steering law selected. The singularity problem is discussed in detail in Ref. 119, and a steering law is presented that is shown to reduce the likelihood of encountering singularities for the case of single gimbal CMGs. Also Ref. 119 establishes the existence of preferred gimbal angles at the beginning of a maneuver that maximize the displacement from the singular configuration.

Also of interest are the host of practical issues that arise due to the fact that real sensors and actuators have dynamics of their own and interact with the structure in nonideal ways, as well as introduce restrictions due to saturation inequalities and bandwidth limitations. Many sensors have significant implicit phase lags that, if ignored, can lead directly to destabilization of an otherwise stable closed-loop system (e.g., even the theoretically robust direct velocity feedback destabilizes modes above the frequency for which the sensor phase lag exceeds 90 deg). We have already mentioned the fact that the commanded control inputs should be consistent with the inputs actually reachable by the actuators (e.g., CMG singularities), but the actuator bandwidth limitations should also be included in the design of the control law.

## IV. Vibration Control for Large Space Systems

### A. Introduction

Here we review methods in the area of vibration suppression for LSS. As suggested in Sec. I, basic tasks are to suppress postmaneuver vibration transients and steady-state vibration-induced line-of-sight (LOS) error and (optical or rf) surface distortions due to steady-state disturbances.

The full technical scope of the LSS vibration suppression problem is very broad. Overall system design and development entails advanced materials and structural design; utilization of passive damping and vibration isolation techniques; advanced methods of structural identification; developments of sensor and actuator hardware, electronics interfaces, and on-board processors; and finally, the development of the *control strategy* as embodied in the algorithm implemented by the on-board digital processor. Although all elements must ultimately be coordinated for successful implementation, this review concentrates on the *control algorithm design* concepts evolved over the past decade in response to the challenges of active vibration control of LSS.

Within the context of overall system development, control algorithm design must perform two separate functions. These may be termed the solution of the "design tradeoffs" and "final design" problems. As an early step in design and a prerequisite to final design, the design tradeoff problem consists in quantitative characterization of the achievable performance of a range of control algorithms under varying hardware capabilities. In other words, the design tradeoff problem is the determination of the fundamental engineering tradeoffs of hardware cost and complexity vs performance. These performance tradeoffs provide the basis for program executive decisions on the selection of control hardware, and control architecture. Then, given the selected actuator, sensor, and processor hardware, the final design problem entails the detailed design of control algorithms meeting precise performance specifications within the limited capabilities of the selected hardware.

Because quantitative tradeoff curves obtained via the design tradeoff activity drive the design/selection of expensive hardware, it is vitally important to ascertain the best achievable performance benefit resulting from a given selection of the cost-impacting hardware design variable. To do this, a suitable control design approach must attempt to determine an "optimal" (in some sense) design subject to the limited capabilities of any one hardware selection. Distinctly suboptimal design can overestimate the extent of hardware cost and complexity needed to achieve performance goals, thus misleading hardware selection decisions.

Since its function is prior to the final design problems and its scope is more comprehensive, the design tradeoff problem has received much of the emphasis of advanced control design concept developments. Specifically, the thrust has been the development of "near-optimal" or optimization-based control design formulations that, to the greatest possible extent, provide the best performance achievable under realistic hardware and processor constraints, i.e., are capable of the most efficient (in the sense described) characterizations of the fundamental engineering tradeoffs. We emphasize herein, the activities supporting this development goal.

The basic technical challenges and the underlying engineering tradeoffs that must be faced by LSS vibration control design are outlined in the following subsection. Here we remark that the severity of the challenge is of so fundamental a character as to strike at the foundations of traditional control theory and practice. In consequence the LSS control design challenge has elicited not only strenuous practical developments but fundamental theoretical advances or the revitalized application of such advances. The abundant technical responses to the LSS control design challenges are reviewed briefly in subsections IV.C–IV.E. We include a synopsis of future research and development needs and a forecast of the technological *opportunities* made possible by these control design advances in Sec. VI.

### B. Major Technical Issues of the Structural Control Design Problem

As just mentioned, LSS control design technology focuses on systematic design methods capable of producing designs of near optimal performance subject to the constraints imposed by limited hardware capability.

First, there are many ways to define "performance." The most frequent measures of performance that have been considered in LSS controls research are 1) mean-square-system response quantities, e.g., rms line-of-sight error and surface error (or mean Strehl ratio for optical systems) and 2) frequency domain performance measures, e.g., disturbance attenuation, return difference singular values, etc.

All of the choices for performance characterization can be meaningful in the appropriate context. The point to note is that the choice of a performance measure is a fundamental defining characteristic of a design theory.

Having chosen a performance characterization scheme, any particular design approach seeks to establish a mathematical framework to choose control parameters to reduce the chosen performance degradation measure as much as possible subject to constraints that reflect a variety of realistic limitations of physical hardware and available processors. There are many such practical constraints.

However, the bulk of LSS controls literature focuses on one or more of the following four fundamental limitations and their associated engineering tradeoffs: 1) limited sensor accuracy and resolution and sensor/electronics noise; 2) limited actuator bandwidth, power, output force (torque), stroke (or angular rate); 3) limited modeling fidelity; and 4) limited space-flight qualified processor computational throughput.

The challenge to LSS control design is to efficiently quantify the tradeoffs of performance vs varying degrees of the four limitations and to do so in the context of a problem involving many inputs and outputs and many structural vibration modes within the control bandwidth.

The presence of densely spaced vibration modes and a large number of inputs and outputs strongly discourages the use of SISO, one-loop-at-a-time design approaches. Thus, the thrust of research in response to the challenge is the development of automated, MIMO design methods.

The earliest attempts to devise MIMO LSS control design approaches used as their point of departure the linear/quadratic/Gaussian (LQG) design theory originated in the 1960s.[120] In the fundamental LQG design formulation, a linear, time-invariant (LTI) plant is presumed to be characterized by a known plant dynamics matrix, control force input matrix, and sensor output matrix. It is supposed that the plant (structure to be controlled) is driven by white noise and that the sensor measurement vector is corrupted by white "observation" noise. The performance "figure of merit" is a weighted sum of steady-state mean-square plant response and mean-square control inputs. The design optimization problem is to choose the control parameter matrices of a dynamic compensator having the same dimension as that of the plant so as to minimize the quadratic performance index. The solution to this problem gives the control matrices in terms of two nonnegative-definite matrices that are determined by solution of two uncoupled (independent) matrix Riccati equations.

By iteratively adjusting the input weighting matrix it is possible to insure that the mean-square actuator outputs are at least in rough conformity with realistic actuator force or torque constraints. Thus LQG offers at least some capability for characterizing the performance tradeoff with respect to actuator limitations (real world limitation 2). By similarly using the intensity of the observation noise as a design variable, it is possible to assess the performance impact of sensor limitations (real world limitation 1). However, the early attempted application of LQG to the LSS vibration control problem revealed its incapacity for meaningful design optimization in the face of limitations in modeling fidelity (real world limitation 3) and in on-line processor computational capacity (real world limitation 4).

A fundamental limitation of LQG for LSS vibration control is its lack of *robustness*; i.e., should the structural plant dynamics as modeled differ (only slightly) from the *actual* plant dynamics (as a result of model parameter errors, modeling computation errors, residual system identification errors or unforeseen in-mission changes) the LQG design predicated on the erroneous model may yield an unstable closed-loop system when finally interconnected with the physical plant. The inherent nonrobustness of LQG was strikingly demonstrated with a simple example by Doyle.[121] The numerous instances of LQG's nonrobustness when applied to LSS vibrational control are illustrated by the performance sensitivity studies in Ref. 122.

A second fundamental limitation is that the dimension of an LQG controller is equal to that of the plant. Since LSS models of reasonable fidelity often contain some hundreds of modes, the LQG controller may place unreasonable burdens on online processing capability. In other words, LQG is limited in its ability to produce compensators conforming with space-flight processor limitations.

Of course, the most straightforward approach to circumvent the LQG dimensionality issue is to delete structural vibration modes in the design model until one has a model of the reduced dimension desired for the controller. But this scheme is thwarted by the "observation and control spillover" problem,[123] whereby corruption of sensor outputs by the unmodeled dynamics and actuator excitation of the unmodeled modes combine to render the closed-loop system unstable. The spillover issue may be viewed as an instance of LQG nonrobustness with respect to errors in the dynamic order of the plant.

In consequence of the given LQG limitations, *robustness* and *controller simplification* have been two dominant themes in LSS controls research over the past decade. There are four categories of control approaches aimed at the treatment of these two issues: 1) direct extensions or modification of LQG theory, 2) methods offering fundamental revision or departure from LQG for the treatment of controller simplification, 3) methods involving fundamental treatment of robustness, and 4) unifying synthesis theories that combine previously disparate points of view to provide simultaneous treatment of robustness and design simplification. Methods in categories 1–4 are reviewed in subsections IV.C–IV.F, respectively.

### C. Direct Extensions of Linear Quadratic Gaussian and Classical Methods

Here we review various approaches to LSS vibration control design synthesis based on application of or direct extension of LQG design and/or classical design principles.

One set of control design approaches aims to circumvent the robustness issues of LQG by exploiting the fact that flexible mechanical structures are inherently passive (energy dissipative) and by configuring control laws that preserve this fundamental property. In this connection an actuator and sensor are said to be *collocated* if there is negligible structural deformation between the actuator and sensor locations. Borrowing from the terminology of classical mechanics, we say that an actuator and a sensor are *conjugate* if a virtual variation in the dynamic quantity measured by the sensor multiplied by the generalized force is identically equal to the rate of virtual work. Thus, an actuator producing a torque along a given axis and a sensor measuring angular rate about that axis are conjugate.

One of the earliest suggested energy dissipative control approaches is the "electronic damping" notion[124] whereby pairs of collocated, conjugate actuators and sensors are combined with a constant, symmetric, positive definite feedback gain matrix to yield an inherently energy dissipative control law which electromechanically emulates passive structural damping. Although simple to design and inherently robust with respect to many structural modeling errors, this scheme entails strict requirements on the bandwidth and frequency response of actuator and sensor hardware since phase shifts due to high frequency actuator and sensor dynamics can destabilize the closed-loop system.

A generalization of the fundamental idea of electronic damping that allows treatment of actuator and sensor dynamics is the "positivity design" technique.[125] Suppose $G(s)$ is some $n \times n$ transfer function matrix. The system represented by $G(s)$ is said to be *positive* (*strictly positive*) if $G(i\omega) + G^*(i\omega) \geq 0$ ($>0$) for all real $\omega$. This condition insures that the system is equivalent to some interconnection of inherently energy dissipative elements. A system composed of two subsystems interconnected in a unity feedback configuration is stable if both subsystems are positive and at least one of them is strictly positive. With collocated and conjugate pairs of actuators and sensors, the structural plant (from generalized inputs to conjugate rate outputs) is at least positive. Thus, one can secure closed-loop stability in the face of structural modeling errors (if these errors do not violate the collocation/conjugate constraints) by constraining the controller to be strictly positive. Note that in the presence of actuator or sensor dynamics and/or sampled data effects an LSS plant is no longer positive. However, positivity results can be generalized to nonpositive systems by operator embedding.

The principal advantage of the positivity approach is that stability is guaranteed in the face of a broad class of modeling inaccuracies and nonlinearities. Cautions to the application of the approach include the need for careful modeling of actuator and sensor dynamics and sampled data effects. Further, the positivity constraint limits the optimization space over which a control design is sought, thereby limiting achievable performance, in conformity with the performance/robustness tradeoffs inherent in control system design. Experimental experiences with application of the method are reported in Ref. 126.

An effort to combine the robustness benefits of energy dissipative control design with the high performance potential of LQG design resulted in the high authority control/low authority control (HAC/LAC) design methodology.[127] The HAC/LAC design comprises a two level controller architecture. The HAC controller is the high-gain part of the controller intended to address the performance oriented goals of the overall design. HAC design is accomplished by applying LQG design theory to a reduced-order model containing the (typically lower frequency) well-modeled, performance significant modes. By this means, HAC/LAC secures a dynamic compensator of appropriately low order. Once the HAC controller is designed, there remains the destabilizing "spillover" interactions with the dynamics not included in the HAC design model. The purpose of the LAC design is to restabilize the residual modes by means of an energy dissipative controller using collocated and conjugate actuator/sensor pairs with constant gain direct sensor feedback. The overall HAC/LAC approach is quite flexible in application except for the need for a subset of collocated and conjugate actuator/sensor pairs. Caution must be exercised to attain accurate modeling information on the HAC-controlled modes since HAC retains LQG nonrobustness with respect to modeling errors within the HAC bandwidth. Also, processor sample rate and sensor and actuator bandwidth must be sufficiently high for successful implementation of the LAC design. Experimental experiences and further conclusions on HAC/LAC design are to be found in Ref. 126.

There is a further set of design approaches obtained by direct extension of LQG or classical methods which attempt to circumvent dimensionality and robustness issues by use of geometric decoupling methods rather than by exploitation of passivity. By far the most straightforward approach in this vein is independent modal space control.[128] The basic idea is illustrated by inspection of the modal coordinate equations of motion

$$\ddot{q} + 2\eta\Omega\dot{q} + \Omega^2 q = bu$$
$$y = c\dot{q}$$

where $\Omega$ and $\eta$ are diagonal matrices containing the frequencies and damping ratios, and $b$ and $c$ are the control input and sensor output model coefficients (and we consider rate sensing only for simplicity). Suppose that the number of sensors and actuators $n$ is sufficiently large that $n$ also is the number of modes needed for acceptable modeling fidelity, thus $b$ and $c$ are full rank. Suppose also that $b^{-1}$ and $c^{-1}$ exist. Then, one can precondition the actuator inputs by setting $u = b^{-1}u'$ and the sensor outputs, setting $y' = c^{-1}y$. Then the system from the new, synthetic control inputs $u'$, to the synthetic sensor outputs $y'$ breaks up into $n$ independent loops, each loop containing only one structural mode. Each independent SISO control design may be accomplished via any one of the applicable SISO design methods to arrive at the final control law of the form $u = b^{-1}G(s)c^{-1}y$. Obvious advantages are that spillover issues are reduced and design computations are greatly simplified. Limitations include the potential requirement for large numbers of sensors and actuators in order to make $b$ and $c$ full rank. Also note that the physical actuator output is $u = b^{-1}u'$, not $u'$. Thus actuator placement must be carefully selected to render $b$ well conditioned to prevent unacceptably large actuator commands. A similar remark pertains to sensor placement to avoid excessive impact of sensor noise.

A more sophisticated geometric decoupling approach which was intended as a technique of reducing control and observation spillover problems is model error sensitivity supression (MESS[129,130]). The fundamental idea of MESS, as explained in Ref. 130, can be simply illustrated by reference to the already given notation. Let

$$b = \begin{bmatrix} b_p \\ b_r \end{bmatrix}$$

where subscript $r$ denotes the residual modes, the spillover effects of which are to be suppressed. Suppose one performs a singular-value decomposition of $b_r$ and constructs matrix $T_B$ with columns proportional to the right singular vectors associated with zero singular values of $b_r$, so that $b_r T_B$ vanishes. Then a conditioning of $u$ via $u = T_B u'$ yields $b_r u = 0$ so that control excitation of the residual dynamics is nulled. By a similar device, one can construct synthetic sensor channels $y'$ that are uncorrupted by the residual dynamics. The approach then applies LQG design techniques to design the controller of the primary dynamics from synthetic inputs $y'$ to control outputs $u'$. In place of geometrically constraining control inputs and sensor outputs, an alternative approach to obtaining the same spillover suppression effects is to apply quadratic optimization to the primary dynamics with the cost function augmented by penalty function terms penalizing observation and control spillovers. In addition, MESS has been extended to filter accommodated model error sensitivity suppression (FAMESS). The filter accommodation part of FAMESS is an attempt, via low pass filtering to prevent destabilization of the less well-known high-frequency modes, beyond the frequency band of the residual dynamics.

When structural mode shape information is accurate and actuator/sensor numbers and placement are appropriate, MESS very effectively suppresses spillover effects. Note, however, that the column dimension of $b_r$ must exceed its row dimension so that the number of actuators must be larger than the number of modes to be decoupled from the controller. Moreover, care must be taken to insure that $b_p T_B \neq 0$ and that the primary modal dynamics are controllable. The satisfaction of these constraints entails requirements for a minimum number of actuators and sensors which increases in proportion to the number of residual modes. Thus, successful application of MESS places demands on sensor and actuator hardware numbers and placement and on mode shape modeling information. Finally, by virtue of its decoupling action, the effective number of independent sensor and actuator channels is reduced and a corresponding penalty on achievable performance is thereby incurred.

### D. Fundamental Approaches for Controller Simplification

For the purpose of obtaining LSS control laws that can be conveniently hosted by available space-flight qualified processors, many design methods have been developed for controller simplification. These methods involve fundamental extension of LQG and classical concepts and generally do not entail, for their application, the imposition of a priori requirements on sensor and actuator hardware. The methods discussed here reveal the possibility of substantial controller simplification without such hardware requirements and thus represent progress toward more efficient characterizations of the performance/controller simplification tradeoff.

Dynamic controller simplification methods may be placed in three general categories: 1) model reduction, 2) indirect controller reduction, and 3) direct controller reduction. In each case it is presumed that open-loop model reduction methods (such as are discussed in Sec. II) are applied to the originally very high-order (typically finite element based) LSS model to arrive at a smaller but still high-order design model of a dimension that can be addressed by available Riccati equation solvers. This design model includes both the "primary dynamics" that are critical to performance and the "residual dynamics" including vibration modes likely to interact significantly with the controller.

Given the high-order design model, model reduction approaches[131] to controller simplification proceed by a further model reduction to the dimension desired for the controller followed by LQG design. Model reduction methods specifically geared to address control performance issues[131,132] have been developed. However, note that an approximation is made early in this process and modeling information is given up leading to potential instability and loss of performance. In consequence, it is generally accepted that approaches that delay (or forego) order reduction approximations are more satisfactory.

Such a class of approaches are the "indirect controller reduction" methods. These approaches, reviewed in Refs. 133–136 proceed by first designing a full-order LQG controller for the high-order design model followed by reduction of the controller dimension, where the controller reduction step may be based on a variety of "figures of merit" characterizing the degree of performance degradation relative to the full-order LQG controller.

"Direct" controller reduction methods essentially forego the approximation steps inherent in the other two approaches by addressing the optimal fixed-order dynamic compensation problem, namely, given a quadratic performance index and a high-order design model, constrain the controller dimension and then determine the gains of the fixed-dimension compensator to minimize the quadratic performance. This is identical to the LQG optimization problem except that the controller dimension $n_c$ is constrained to be less than $n$.

We first describe work on the direct approach because the results reveal the essential structure of the controller reduction problem and provide a unifying framework for discussion of the (large number of) indirect reduction methods. Early efforts on this direct approach (for example, Refs. 137 and 138) proceeded by computation of the performance gradients combined with gradient search methods to converge to solution of the constrained optimization problem. Later, the optimal projection (OP) theory, introduced in Refs. 139 and 140, arrived at analytical formulation of the first-order necessary conditions for the constrained optimization problem. The OP approach is to solve the OP design equations to obtain the extrema and then to evaluate the performance of each extremal design to determine the one corresponding to the global minimum.

The OP results for the first-order necessary conditions reveal the basic structure of the optimal reduced-order compensator and provide insight into indirect controller reduction methods. The OP first-order necessary conditions consist of LQG-like expressions for the compensator gains that are further determined by four design equations: modified regulator and observer Riccati equations and two "pseudogrammian" modified Lyapunov equations. All four design equations are coupled by a projection (idempotent matrix) that describes the geometric structure of the reduced-order controller.

The OP *design* approach, as distinct from fixed-order controller characterization results, is based on the converse to first-order necessary conditions. The optimal reduced controller must satisfy the OP conditions; and conversely each admissible solution (there is, in general, more than one) to the OP equations is an extremal; and *one* of these extremals is the global minimum. Thus the direct OP design approach is to compute several solutions, then select the solution having the smallest closed-loop cost. Each admissible solution results in a stable closed-loop compensator. A variety of computational methods are given in Refs. 134, 135, 141, and 142. The most effective is the homotopy approach given in Ref. 142 which also allows the desired OP solution to be preselected by choice of the homotopy starting solution. Experimental experiences in the application of the optimal projection design approach to flexible structures are reported in Refs. 143 and 144.

The advantages of the direct approaches is that a controller design of desired order can be obtained whenever it exists and its performance (judged by the selected performance index) cannot be improved upon; i.e., the performance/controller complexity tradeoff is characterized most efficiently. However, convergent OP solution techniques entail advanced computational methods and nonstandard software.

Indirect methods of controller reduction offer computationally simpler techniques that may also give the user some insight into the reduction procedure. The proposed indirect methods are given in Refs. 145–154 and are reviewed in Refs. 133–135 and 148. The basic idea of the indirect methods is to select a figure of merit (not necessarily a quadratic state-space criterion) that characterizes closed-loop performance or, alternately, the performance degradation incurred by controller order reduction relative to the high-order LQG controller, and then to construct a tractable synthesis procedure that is likely to reduce the figure of merit to acceptable levels.

A wide variety of figures of merit have been adopted and incorporated within a large number of proposed methods. These are elucidated in Ref. 133. However, the substantial similarities among the methods is revealed by reducing them to a common notation and using the unifying framework of OP results. This method of comparison was used for several indirect methods in Ref. 134 and reveals that many of the indirect methods are approximations to the optimal fixed-order control design formulation.

For practical applications, an indirect method should not only be computationally tractable; it must also exhibit reasonable reliability in producing closed-loop stable controllers. Whereas at present, no indirect method can rigorously guarantee a stable controller of given order whenever a stabilizing controller of that order exists, success in the majority of cases is desirable. This point has sparked several numerical comparisons and encouraged the invention of refined methods. For example, Refs. 133, 135, 136, and 154 conducted numerical comparisons using an example problem introduced in Ref. 153. These comparisons encompassed five indirect methods and OP. The five indirect reduction methods experience increasing difficulty as controller order decreases and controller authority increases. More recently, an additional improved indirect method involving controller canonical correlation coefficients, method $C^4$, was proposed in Ref. 154. Further comparisons between the $C^4$ method and optimal projection were conducted in Ref. 136. Although $C^4$ exhibited degraded performance tending toward instability, OP designs gave monotonically improving performance. At lower controller authority levels, however, $C^4$ resulted in good approximation to the quadratically optimal performance.

### E. Fundamental Approaches for Modeling Error-Tolerant Control Design Synthesis

Although the development of LQG theory in the 1960s promised a truly multivariable design theory, it eventually became clear that LQG theory had numerous deficiencies in practical application. For example, LQG controllers lacked rudimentary forms of tolerance to plant modeling errors such as gain and phase margins[155] and could be arbitrarily sensitive to parameter variations.[156,157] Such deficiencies were to be expected, however, since an LQG controller is constructed to optimize a single, narrowly defined performance measure, namely, a quadratic cost. Although a quadratic performance measure is physically relevant to many applications such as regulation with rms error specifications in the presence of persistent disturbances, the deficiencies of LQG design sparked fundamental efforts to devise error-tolerant control synthesis methods.

With respect to the toleration of modeling errors, we distinguish *lack of sensitivity* from robustness. Sensitivity refers to the incremental impact on system outputs and performance due to infinitesimal changes in the plant model. On the other hand, robustness with respect to a system property (e.g., stability) means the preservation of that property in the face of finite perturbations and errors. Since recent advances in robust control design for stability robustness represent an important chapter in LSS control technology development, these results are the focus of this subsection.

Basic aspects of the robustness problem may be discerned from the various sources of modeling uncertainty in LSS control design. The structural element of the overall system is characterized by an element of a Hilbert space, evolving according to a damped wave equation. Owing to inaccuracies in the physical description and the discretization procedure the actual inertia, damping and stiffness operators may differ from their nominally modeled counterparts by time-invariant perturbations. Note that these perturbations are highly structured; for example, whatever the perturbations might be, physical realizability constrains the inertia, damping and stiffness operators to have non-negative definiteness properties. Likewise, various input/output relations, for example, the operator relating the state to the physical motions to be measured by sensors, has potential errors characterized by time-invariant, structured perturbations.

The modeling uncertainties of this type are of the class known as *parametric uncertainties*. Uncertainties are parametric when the system is described by a state-space model with specifically defined coefficient matrices but the precise values of the coefficients are not known, except within certain bounds.

Apart from the structural subsystem, most other elements of active control instrumentation (actuators, sensors, and the dynamic compensator) can be subject to both parametric and *nonparametric* uncertainties. Basically, a system subject to nonparametric uncertainty is characterized by bounds (of various types) placed upon the errors in the input-output transfer function characteristics and there need not be postulated any state-space description of the system. In the nonparametric class, the literature distinguishes *unstructured* from *structured* nonparametric uncertainty. For example, one could lump (via rearrangements and transformations) all the uncertain perturbations of the system into a single nonparametric uncertainty inserted at some point in the unity feedback representation. As an alternative to this unstructured nonparametric description, one might further articulate the uncertainties into several blocks occurring at various points in the loop to arrive at a structured, nonparametric description.

The preceding categories of uncertainty description form an ordered sequence moving from the highly articulated, highly refined to the less articulated and less structured descriptions. This ordering of uncertainty classes gives rise to the concept of *conservatism* in robustness analysis. A robustness analysis technique is conservative if the predicted set of stable perturbations is a proper subset of the actual set of stable perturbations. The impact of conservatism is essentially to give up useful system information, resulting in unnecessarily pessimistic predictions and overdesign of control systems.

The historical development of robustness theory has moved from the cruder, less structured uncertainty descriptions toward consideration of more highly articulated uncertainty sets. In part this movement was impelled by conservatism issues. The result, at present, is a suite of analysis and design tools covering the full range of uncertainty types.

Motivated by the deficiencies of the LQG formulation the earliest robustness analysis approach considered unstructured, nonparametric uncertainties.[158] Robust stability conditions involve inequalities on the weighted singular values of the return difference operator. Corresponding to this singular value analysis, a design synthesis approach was devised which hinges on the result that a quadratically optimal full-state feedback (LQR) controller has 6 dB/60-deg gain and phase margins.[159] However, full-state feedback controllers are only implementable in the case wherein the system output matrix is the identity matrix.

Such restrictions notwithstanding, however, these LQR features suggested a remedy to the LQG defects: speed up the regulator and/or estimator dynamics to "recover" the LQR properties.[158,160] This approach is a specific application of a more general class of loop shaping procedures ultimately aimed at allowing performance/robustness tradeoffs.[161,162] By minimizing sensitivity at low frequency and complementary sensitivity at high frequency, it becomes possible to achieve both performance specification and robustness to unmodeled dynamics. Note that these approaches properly pertain only to unstructured uncertainty and proportional input or output perturbations. With parametric uncertainty and nonproportional perturbations, these techniques can be arbitrarily nonrobust.[156,157]

Such loop shaping procedures were, however, predicated on a quadratic performance criterion. Thus, there were two drawbacks: 1) since the quadratic criterion concerns only the $H_2$ norm of the frequency response, the designer has only limited control over the loop shape, and 2) since the description of plant uncertainty is not consistent with the performance metric, it is difficult to derive robust performance bounds. Both difficulties were eventually overcome with the development of $H_\infty$ control theory.[163,164] In $H_\infty$, the system is described as an element of a Hardy space with $L_2$ (bounded power) inputs and outputs. The induced norm is the $H_\infty$ norm, which is the supremum over $\omega \in [-\infty, \infty]$ of the maximum singular value function of the system transfer matrix. In contrast to the LQG quadratic performance measure (equivalent to the $H_2$ norm), the $H_\infty$ performance measure corresponds to worst-case frequency attenuation, i.e., if $G_p(s)$ is the transfer function from inputs to performance related variables, then $H_\infty$ design chooses the controller to minimize

$$\|G_p\|_\infty \triangleq \sup_\omega \bar{\sigma}\left[G_p(j\omega)\right]$$

Because of these features, $H_\infty$ permits precise loop shaping and uncertainty characterization by means of dynamic weights. Extension of $H_\infty$ synthesis to a state-space formulation and the recent development of mixed $H_2/H_\infty$ optimization are discussed in the next subsection.

When one is faced with structured nonparametric uncertainties, the previous techniques may be unduly conservative. First note[165] that any system with several conic sector perturbations inserted at several points in the loop can be recast into a block diagonal perturbations diagram. One could ignore the block diagonal structure and apply the small gain theorem to obtain the singular value (SV) robust stability condition. However, because the structure of the nonparametric perturbations is ignored, this result can be arbitrarily conservative. It was precisely in order to remedy such conservatism in the face of structured perturbations, that the structured singular value (SSV) was introduced.[166] The SSV or "$\mu$-function" leads to the "small $\mu$ theorem" which generalizes the small gain theorem for structured nonparametric uncertainties. Methods for SSV computation are given in Ref. 167 and analytical examples for SSV robustness analysis are treated in Ref. 168.

SSV design synthesis incorporates frequency domain performance specifications by inclusion of fictitious uncertainty blocks. For synthesis purposes $\mu$ has usually been computed via the upper bound expression given in Ref. 166. This upper bound is essentially an optimally, diagonally scaled $H_\infty$ norm so that optimization proceeds along $H_\infty$ lines. Analytical examples are found in Ref. 168 and experimental experiences are reported in Ref. 169.

Although SSV analysis and synthesis incorporated both frequency domain performance specifications and robustness with respect to structured nonparametric uncertainties, it (and $H_\infty$ design) does not simultaneously address controller simplification. In many instances the dimension of SSV designed (and $H_\infty$ designed) compensators is considerably larger than the plant dimension, and additional controller reduction steps must be carried out. Also, although in principle SSV may be formulated to handle real parameter variations, there are serious computational problems of a combinatorial nature and this aspect is not yet mature enough for LSS applications.

Just as unstructured uncertainty robustness analysis methods can be arbitrarily conservative when applied to structured nonparametric perturbations, both structured and unstructured nonparametric methods are conservative with respect to parametric uncertainties. This is illustrated by the simple example of a single degree-of-freedom oscillator (a single structural mode) with uncertain stiffness and an idealized rate feedback controller. If we represent the stiffness uncertainty by a conic sector perturbation (ignoring the fact that the perturbation is real valued) both SV and SSV analysis yield the same robust stability condition, which states that robust stability prevails if the relative variation of the stiffness is no larger than the closed-loop damping. However, the system is obviously stable for all positive values of the stiffness. Thus, this robustness condition can be extremely conservative for lightly damped structures.

To obtain less conservative results, many robustness analysis methods explicitly addressing parametric uncertainties have been developed. To illustrate the basis for a set of these methods, consider systems of the following form:

$$\dot{x} = (A + \Delta A) + Dx, \qquad z = Ex, \qquad R \triangleq E^T E$$

where $A$, $D$, and $E$ are fixed matrices, $z$ is the system output vector, $w$ is a white noise, and $\Delta A$ is constrained in some parametered set $\mathcal{U}$.

For the preceding system, we can give a unified description of "guaranteed cost" methods. The quantity

$$\mathcal{J}(\mathcal{U}) \triangleq \sup_{\Delta A \in \mathcal{U}} \lim_{t \to \infty} E\left[z^T(t) z(t)\right]$$

represents the worst-case quadratic performance. Guaranteed cost methods proceed by constructing a "bound function," $\Omega(Q)$

$$\Omega(Q) \geq \Delta A Q + Q \Delta A^T \quad \text{for all} \quad \Delta A \in \mathcal{U} \quad \text{and} \quad Q \geq 0$$

This is then used to form the "auxiliary" second moment equation

$$0 = AQ + QA^T + \Omega(Q) + V$$

which serves as the basis for design optimization, i.e., control design parameters are chosen to minimize the auxiliary cost, $\mathrm{tr}[QR]$. The key result is that once design synthesis is accomplished such that $Q$ exists satisfying the auxiliary second-moment equation, then for all $\Delta A \in \mathcal{U}$, $(A + \Delta A)$ is asymptotically stable and $\mathcal{J}(\mathcal{U}) \leq \mathrm{tr}[QR]$. Thus this general approach guarantees some level of worst-case performance (the guaranteed cost) despite modeling uncertainty. Since the ordering induced by the cone of non-negative definite matrices is only a partial ordering, there may not exist a bound function $\Omega(\cdot)$ that is a least upper bound. Thus there are many alternative definitions for $\Omega(\cdot)$ and each choice gives rise to a distinct guaranteed cost method.

The preceding idea gives rise to a variety of methods each associated with a particular choice of the bound function $\Omega(\cdot)$. The absolute value bound was among the first methods proposed in this vein.[170,171] Associated design synthesis methods necessarily approximate optimization with respect to $\mathcal{J}(\mathcal{U})$ since, in this case, $\Omega(Q)$ is nondifferentiable. The quadratic bound treated in Refs. 172–174 is of great theoretical interest since it is a pivotal result in the unification of $H_\infty$ and $H_2$ points of view as will be described in the next subsection. The linear bound[172,175] leads to the most convenient auxiliary second-moment equation for the purpose of optimal design synthesis formulation and has interesting stochastic interpretations.[176]

The preceding three bounds essentially complexify the parametric uncertainties whereas the maximum entropy (ME) theory[177–179] is an example of formulations specifically addressing real-valued parameter uncertainties. This bound is so named because it was initially formulated from application of information theoretic principles. Briefly, the ME bound is constructed so that not only does $\Omega(Q)$ overbound the perturbation terms but also the solution to the $Q$ equation gives an overbound to the a priori system information lost in consequence of parameter uncertainty. The a priori information lost is measured by the information theoretic *entropy* of the system outputs. ME modifications to the second moment equation resemble terms arising in Stratonovich multiplicative noise models.[180] References 179 and 181 give analytical examples of maximum entropy design synthesis results. Experimental experiences in the application of ME to LSS are reported in Refs. 182–184.

### F. Coalescences: Unifying Synthesis Formulations

Basic advances treating controller simplification and robustness have been described in the preceding section. These fundamental approaches arose from diverse points of view. Recently, however, rapid progress has been made in unifying and reconciling the apparently diverse approaches just described. These developments have had immediate impact on practical applications of modern control design tools to LSS and so are reviewed here.

There are two lines of coalescence: approaches to the amalgamation of robustness theory with controller simplification and the rapproachment between $H_2$ performance-based theories and the treatment of frequency domain specifications via manipulation of $H_\infty$ performance constraints.

The first category of unifying approaches are exemplified by those which directly combine fundamental controller reduction theories with the fundamental robustness revisions of LQG for handling structured and parametric uncertainties. For example, Refs. 172 and 174 describe the fundamental elaboration of optimal controller simplification theory to systems having uncertainties described by the various $\Omega$ bounds just reviewed. These results give optimality conditions (to be solved in design synthesis) for fixed-order control design in the presence of uncertainties characterized by the selected $\Omega$ bound.

A second major coalescence of previously disparate theoretical streams is the rapproachment of the state-space/$H_2$ performance with frequency domain/$H_\infty$ perspectives in the form of mixed $H_2/H_\infty$ theories. There are two principal motivating factors for this coalescence. First, although quadratic performance measures are directly meaningful for LSS application, the imposition of frequency domain disturbance attenuation specifications is often necessary in practical design. This encourages the fundamental reconciliation of $H_2$ and $H_\infty$ points of view on optimal design synthesis. Another incentive for coalescence arises from the fact that $H_\infty$ optimization computations lacked the elegance of Riccati equation solutions as in LQG, thus a state-space formulation of $H_\infty$ was desired.

Within the $H_\infty$ Riccati equation approach, synthesis results for full-state feedback first appeared in Ref. 185 with extensions in Refs. 186 and 187. Synthesis theory for the practical case of dynamic compensation appeared simultaneously in Refs. 188 and 189, where Ref. 188 proved sufficient conditions for $H_\infty$ optimal (for the "standard problem") fixed-order compensation and Ref. 189 demonstrated necessary and sufficient conditions for full-order compensation. When the design synthesis equations in Ref. 188 are specialized to equalized $H_2$ and $H_\infty$ weights and full-order compensation ($n_c = n$), the equations of Ref. 189 are recovered as a special case.

The basic technique by which the $H_\infty$ standard problem is reduced to a state-space formulation[188] is quite simple. Rather than solve either the $H_2$ or $H_\infty$ problems, one replaces the second-moment Lyapunov constraint equation in the $H_2$ design problem by an "auxiliary," second-moment equation that is modified by a quadratic term with a weighting constant $\gamma$. Then, similar to the mechanisms involved in the $\Omega$-bound results discussed earlier, the second-moment matrix and the quadratic cost of the $H_2$ design are both bounded by the results of the auxiliary minimization problem; and the $H_\infty$ norm of the closed-loop transfer function is bounded by $\gamma$. If $\gamma$ is allowed to increase indefinitely, then the results of the $H_2$ design formulation are recovered. On the other hand (as shown in Ref. 189), if $\gamma$ is progressively reduced to the minimum value for which the auxiliary second-moment equation possesses a solution, then one obtains the solution to the $H_\infty$ optimization problem. An important additional feature is that by exploring the continuum of intermediate values of $\gamma$, one can strike any desired tradeoff between optimal $H_2$ and optimal $H_\infty$ designs. This mixed $H_2/H_\infty$ formulation thus achieves a fundamental reconciliation between $H_2$ and $H_\infty$ world views.

Finally, the mixed $H_2/H_\infty$ approach is fully amalgamated with controller simplification by the extensions of the auxiliary minimization problem to include fixed-order compensation constraints given in Ref. 188. In summary, the recent unifying synthesis theories offer a rather complete range of design tools for LSS control synthesis and potentially constitute a successful response of the control systems community to the technical challenges implicit in the limitations of both classical and LQG design.

## V. Hardware Developments and Experiments

The challenges to advanced system identification and control technologies were described in the Introduction and Sec. IV.A. Sections II–IV surveyed the responses to these challenges, putting emphasis on theoretical/algorithmic developments since the bulk of the literature deals with this area. However, for sound, overall development of the new technologies, hardware implementation and verification via ground-based and space-flight experiments are essential.

Even with regard to theoretical developments, such experimentation serves a two-fold purpose. First of all, they can validate (or even invalidate) existing theories proposed for the control of flexible structures. Second, the insights gained from actual hardware experiments elucidate important issues in practical control design and thus provide needed direction for future theoretical research.

In addition to validating and guiding theory, experimentation also highlights the primary technology issues involved in implementing active control for flexible structures. For example, experiments tend to reveal the importance of sensor and actuator development tailored for flexible structures. Also, the number of design iterations required in the control design portions of these same experiments and the large amounts of data manipulation reveal the need for the continual evolution of efficient design methodologies.

The needs for experimentation have stimulated many ground-based experiments that have been conducted over the past decade or are presently being planned near term. On the topic of experimentation several excellent and detailed reviews are available[190-192] particularly on pre-1982 experiments involving relatively simple, monolithic structures such as beams and plates. More recently Ref. 193 thoroughly reviewed post-1982 experiments, including experiments involving more complex structural configurations, experimental efforts that have progressed "full term" to produce reportable experimental results, and planned experimental facilities that can be expected to produce such results near term. Because of the wealth of details given[193] we refrain from repeating this material here. Instead, we merely highlight some of the overall trends and discuss a few examples illustrating successful application of structural vibration control.

Many of the experiments discussed in Ref. 193 have resulted from building on previous experiences obtained through the numerous simpler experiments reviewed in Refs. 190 and 191 and display a significant degree of sophistication. Even the simpler of the post-1982 test beds are relatively complex assemblages of more basic structures and thus exhibit high modal density and dynamic complexity. The most advanced test facilities not only replicate the generic characteristics of LSS dynamics and control problems but also are directly traceable to particular LSS design concepts. By "traceable," it is meant that the structural and control components of an experimental test bed stand in an approximately one-to-one correspondence with the principal structural/control elements of a projected LSS concept. Thus, the exercise of a traceable experiment provides a kind of "dress rehearsal" of the new technology as it might be implemented aboard a real flight system. Of course, no one traceable facility can simultaneously address all aspects of LSS control technology and each test bed must focus on some particular aspect. Nevertheless, the emergence of traceable experimental facilities is a significant sign of the maturation of LSS technology over the past decade. For example, vibration suppression technology for the large truss beams that will form the principal structural components of future LSS such as rf systems, Earth observation platforms, and Space Station is addressed by the Mini-Mast Facility,[194] which involves a full-scale structural component. At a more comprehensive system level, the ACES facility[195] and CSI evolutionary model[196] are traceable test beds addressing large offset fed antenna systems or Earth observation platform concepts, respectively. Finally the ASTREX facility[197] and MHPE,[198,199] represent traceable test beds for large multisegment reflectors, focusing on 1) rapid retargeting with LOS jitter control and 2) vibration suppression and dynamic surface shape control, respectively.

Another trend to be noted is that the more recent test beds show a progressive widening of technical scope to address more and more issues simultaneously. Thus, many of the experiments whose primary focus is given as "vibration suppression" are actually configured to simultaneously address system identification and a variety of control design challenges. A notable element of overall progress in this field has been the emergence of "guest investigator" efforts which provide several investigative groups the opportunity to conduct multidisciplinary research on traceable test beds that would otherwise be beyond the technical and financial means of individual researchers. The longest standing and, to date, the most productive and effective effort of this kind is the NASA controls-structures interaction (CSI) guest investigator (GI) Program. In its first phase eight research groups were contracted to work on two NASA facilities (the Mini Mast and ACES facilities) to research a wide variety of topics including modeling, system identification, line-of-sight pointing, vibration suppression, and fault detection. The more recent follow-on program[196] similarly addresses a broad selection of issues using an expanded selection of test facilities.

Beyond the conception and implementation of advanced test facilities, the most important element of progress has been the acquisition of real experimental results that illustrate the power and effectiveness of advanced system identification and control design technologies for LSS. Since such results strongly corroborate the system development opportunities to be gained via use of advanced LSS control technology, we conclude this section by brief descriptions of two examples drawn from the NASA CSI GI program.

The first example involves the Mini-Mast facility at NASA Langley Research Center (LaRC) which is traceable to the large truss beams substructures of several LSS concepts. The Mini-Mast is a vertically oriented, 20-m-long, 18-bay truss beam cantilevered at the base with reaction wheels on the tip platform, beam mounted accelerometers at various stations, and position sensors for displacement monitoring. The typical control objective is to damp the translation of the beam tip in response to shaker disturbances introduced at bay 10 (approximately at midspan). In pursuit of this control objective the selected control design for this example (see Ref. 200) used the torque wheels and only the beam mounted accelerometers at the tip platform and bay 10 to implement a discrete-time dynamic compensator. Thanks to the accuracy of finite element and ERA-generated models provided by NASA LaRC, the control design was implemented and tested at LaRC at a single test session with flawless adherence to the test plan and with closed-loop results virtually identical with pretest predictions. Figure 1 shows comparison of the analytical prediction and test data for the tip response due to a midspan shaker impulse disturbance. The obvious agreement illustrates the powerful advantages offered by advanced system identification methods to effective and highly predictable active vibration control. At the same time the approximately 50% first mode closed-loop damping ratio illustrates the efficacy of advanced active control methods for this structural configuration.

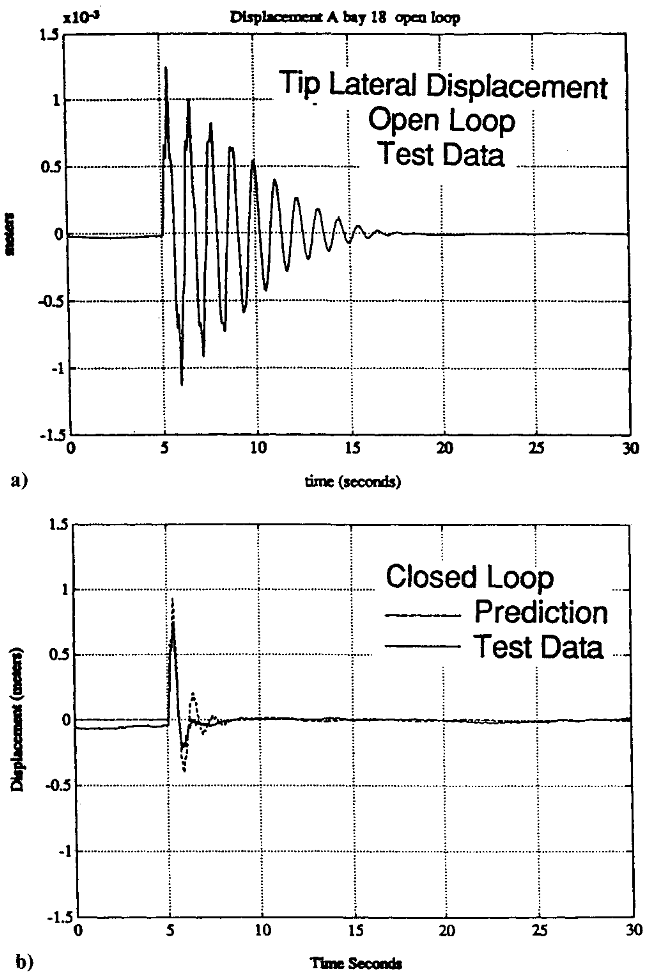


**Fig. 1 Mini-Mast: open- vs closed-loop performance a) tip lateral displacement, open-loop test data and b) closed-loop prediction vs test data; in both cases, excitation is a shaker pulse disturbance.**

A second example, involving the ACES experimental test bed, shows the further amalgamation of system identification and control design into a more nearly unified design/test methodology and demonstrates vibration control for a test bed that is traceable to a large offset-fed antenna system. The main structural component of ACES, a spare Voyager Astromast, is a lightly damped truss beam approximately 45 ft long. The total configuration consists of an antenna and counterweight legs appended to the Astromast tip and the pointing gimbal arms at the Astromast base. The overall structure contains many closely spaced, low-frequency modes (more than 40 modes under 10 Hz).

The goal of the control design is to position the laser beam in the center of the optical detector. The example control design discussed here (see Ref. 201 for details) used eight control inputs and eight measurement outputs. The inputs were the image motion compensator (IMC) gimbal torques (two axes), advanced gimbal system (AGS) torques (two axes), and the $x$ and $y$ axes forces of two linear momentum exchange device (LMED) packages. Measurements consisted of $x$ and $y$ detector (DET) position outputs, $x$ and $y$ base gyro (BGYRO) rate outputs, and the $x$ and $y$ outputs of the LMED accelerometers. The disturbances to the system were position commands to the base excitation table (BET).

In the example considered here, both system modeling and control design were performed by the same investigative group. Specifically, ERA was used to generate state-space models from open-loop test data. The excellent agreement in both magnitude and phase permitted direct control design via an advanced fixed-structure control design method. The resulting controller design was a decentralized control consisting of three relatively low-order dynamic compensators having 6, 10, and 12 states, respectively. Figure 2 shows the experimental results for the LOS response to a BET pulse input for both open and closed loop. The nearly three orders of magnitude reduction of the LOS bias and order-of-magnitude reduction in vibration decay time illustrate the effectiveness of advanced control methods in addressing LSS performance issues.

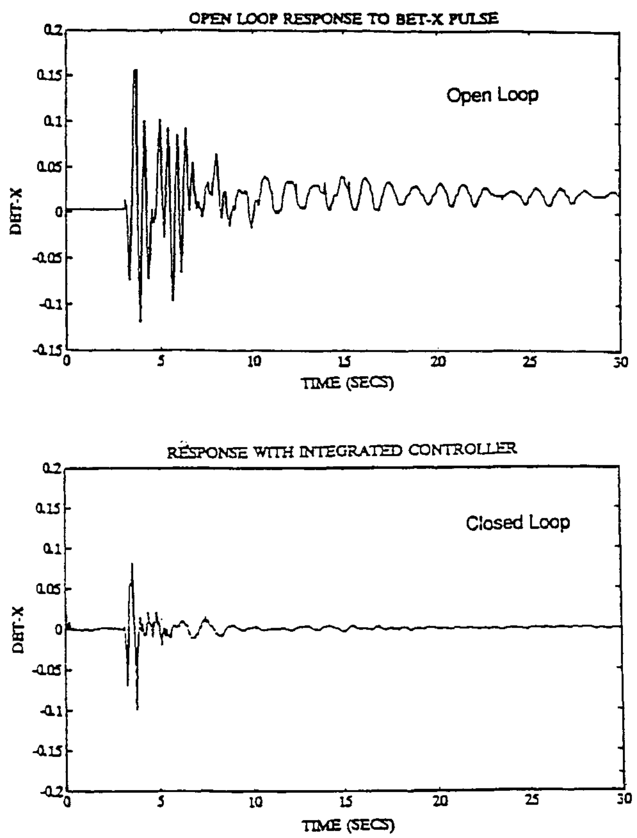


**Fig. 2 Example results for ACES test-bed: open-loop vs closed-loop LOS-X performance with the controller of Ref. 201.**

## VI. Future Needs and Directions

The development of active control technology for LSS over the past decade has stimulated significant advances in design theory and has made great strides toward applicability. However, much remains to be done for the maturation of LSS control technology into an area of professional competence backed by reliable, effective engineering design tools. Areas requiring further efforts include theoretical issues connected specifically with the structural control application and efforts to transition theory into practice via the use of ground-based test beds and space-flight experiments.

With regard to theoretical efforts, controller simplification in the context of robust design remains an active area of fundamental research. At minimum there is need for further elaboration and/or consolidation of the newer unifying approaches discussed in Sec. IV.

Currently, a paramount issue in robust control design for LSS is concerned with the role of real parameter uncertainties and phase information. We briefly describe the nature of this issue via analogy to classical design concepts.

From a classical control-design point of view, the issues of real parameter uncertainty and phase information are manifested in the fundamental concepts of gain and phase stabilization. In terms of gain stabilization, stability of a SISO, closed-loop system is insured by designing the controller so that the magnitude of the loop transfer function is less than unity in frequency regimes in which the phase is either known to be near 180 deg or is highly uncertain. In terms of phase stabilization, stability is achieved by insuring that the phase of the loop transfer function is well behaved where the loop transfer function has gain greater than unity. Roughly speaking, phase stabilization can by used to allow high loop gain and thus achieve high performance in frequency regimes in which sufficient phase information is available, whereas gain stabilization (e.g., rolloff) is needed to insure stability where the phase of a system is very poorly known.

In the context of multivariable design for LSS phase considerations arise particularly in connection with structural modeling uncertainties in the mass and stiffness matrices. These are in the class of energy preserving perturbations discussed earlier, for which open-loop stability is maintained for any perturbation in this class. In terms of phase information, any set of energy preserving perturbations will still preserve the phase of force input to collocated rate measurement transfer functions within from $+90$ to $-90$ deg. However, most of the robustness methods discussed, e.g., all those based on norm bounds and nearly all the guaranteed cost methods, are very conservative with respect to real parameter uncertainties especially the energy preserving class. This is because such methods effectively treat parametric uncertainties in mass and stiffness matrices as complex, unstructured perturbations (so that a stiffness uncertainty looks like a large damping uncertainty). The result, for LSS, is overcompensated design calling for unrealistically high actuator effort. In summary, using analogies to classical design, most robustness methods so far devised succeed in generalizing the classical concept of gain stabilization to the multivariable case, but the corresponding generalization of phase stabilization remains to be accomplished. Both analytical and experimental experience indicates that this deficiency is a serious roadblock to the development of truly effective design tools.

To help in the transition of theory into practice, there is need for continuing experimentation on ground-based test beds. It is encouraging that the needs for experimentation have stimulated the many ground-based experiments briefly surveyed in the previous section. A particular element of overall progress in this field has been the emergence of guest investigator efforts which provide several investigative groups the opportunity to conduct multidisciplinary research on traceable test beds that would otherwise be beyond the technical and financial means of individual researchers. Because of their productivity, we strongly recommend the continuation and expansion of such guest investigator experimental activities.

Finally, with such a variety of ground-based experiments the key question is: Why are space-flight experiments needed? An immediate and fairly obvious answer is that they are needed to follow up on and perform final validation of the technology demonstrations accomplished in ground-based experiments. A more profound, and perhaps, compelling answer, however, is that a flight experiment is needed to address a variety of interdisciplinary, interactive issues that are neither well addressed in current ground experiments nor can be satisfactorily and cost-effectively addressed by current or projected ground tests.

Current ground experiments each address a relatively focused technology area, but not the integration under realistic conditions of several such areas. However, the enforced realism of a flight experiment will drive us to grapple with a host of interactive issues. Two of these that particularly touch on control design technology are as follows.

1) Deployable space structure design: The dimensions of current or projected launch systems severely limits the size of nondeployable payloads. Implementation of planned LSS requires deployable structure design wherein the structure assumes a compact stowed configuration and is then unfolded and locked into a much larger on-orbit configuration. Deployability considerations ripple throughout the system design, influencing many issues, e.g., dimensional tolerances and repeatability, the need for unobtrusive actuators and sensors, etc.

2) Integrated system design and performance analysis of large multicomponent systems: This issue arises from the fact that a large flight system has many, many subsystems and control systems. The challenge is not only to develop and implement the component controllers, but also to systematically design and evaluate the large multicomponent, multicontroller system.

The interdisciplinary, interactive issues can be, and for the most part are, avoided by ground-test programs, because they are necessarily focused on strictly delimited technology areas. Nevertheless, these issues must be faced and solutions must be validated for future development of LSS to be possible. Only the enforced reality of a flight experiment can stimulate the technical community to successfully grapple with the interactive issues. Moreover, only the physical capabilities of a flight experiment can convincingly demonstrate the resolution of these issues.

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